Edexcel Maths FP3

Topic Questions from Papers

Vectors

2.

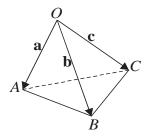


Figure 1

The points A, B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively, relative to a fixed origin O, as shown in Figure 1.

It is given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j}$$
, $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

Calculate

(a) $\mathbf{b} \times \mathbf{c}$,

(3)

(b) $a.(b \times c)$,

(2)

(c) the area of triangle OBC,

(2)

(d) the volume of the tetrahedron *OABC*.

(1)

Question 2 continued	l l
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The lines l_1 and l_2 have equations

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \text{ and } \mathbf{r} = \begin{pmatrix} \alpha \\ -4 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}.$$

If the lines l_1 and l_2 intersect, find

(a) the value of α ,

(4)

(b) an equation for the plane containing the lines l_1 and l_2 , giving your answer in the form ax + by + cz + d = 0, where a, b, c and d are constants.

(4)

For other values of α , the lines l_1 and l_2 do not intersect and are skew lines.

Given that $\alpha = 2$,

(c) find the shortest distance between the lines l_1 and l_2 .

(3)

Question 7 continued	b



The plane Π has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \lambda (-4\mathbf{i} + \mathbf{j}) + \mu (6\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

(a) Find an equation of Π in the form $\mathbf{r.n} = p$, where \mathbf{n} is a vector perpendicular to Π and p is a constant.

(5)

The point P has coordinates (6, 13, 5). The line l passes through P and is perpendicular to Π . The line *l* intersects Π at the point *N*.

(b) Show that the coordinates of N are (3, 1, -1).

(4)

The point R lies on Π and has coordinates (1,0,2).

(c) Find the perpendicular distance from N to the line PR. Give your answer to 3 significant figures.

(5)

Question 7 continued	blank

6. The plane P has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$$

(a) Find a vector perpendicular to the plane P.

(2)

The line l passes through the point A(1, 3, 3) and meets P at (3, 1, 2).

The acute angle between the plane P and the line l is α .

(b) Find α to the nearest degree.

(4)

(c) Find the perpendicular distance from A to the plane P.

(4)

Question 6 continued	blank

3.	The position vectors of the points A , B and C relative to an origin O $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $7\mathbf{i} - 3\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j}$ respectively.		ank
	Find		
	(a) $\overrightarrow{AC} \times \overrightarrow{BC}$,	(4)	
	(b) the area of triangle ABC,	(2)	
	(c) an equation of the plane ABC in the form $\mathbf{r.n} = p$	(2)	
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		_	

Question 3 continued	bla

2. Two skew lines l_1 and l_2 have equations

$$l_1: \mathbf{r} = (\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$$
$$l_2: \mathbf{r} = (3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) + \mu(-4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$$

respectively, where λ and μ are real parameters.

(a) Find a vector in the direction of the common perpendicular to $\boldsymbol{l_1}$ and $\boldsymbol{l_2}$

(2)

(b) Find the shortest distance between these two lines.

(5)

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8. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1

(3)

The plane Π_2 has vector equation

 $\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$, where λ and μ are scalar parameters.

(b) Find the acute angle between Π_1 and Π_2 giving your answer to the nearest degree.

(5)

(c) Find an equation of the line of intersection of the two planes in the form $\mathbf{r} \times \mathbf{a} = \mathbf{b}$, where \mathbf{a} and \mathbf{b} are constant vectors.

(6)

Question 8 continued	blan
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TOTAL FOR PAPER: 75 MARK	.S
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Further Pure Mathematics FP3

Candidates sitting FP3 may also require those formulae listed under Further Pure Mathematics FP1, and Core Mathematics C1–C4.

Vectors

The resolved part of \mathbf{a} in the direction of \mathbf{b} is $\frac{\mathbf{a.b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\frac{\mu \mathbf{a} + \lambda \mathbf{b}}{\lambda + \mu}$

Vector product:
$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \, \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

$$\mathbf{a.(b\times c)} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \mathbf{b.(c\times a)} = \mathbf{c.(a\times b)}$$

If A is the point with position vector $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ and the direction vector \mathbf{b} is given by $\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$, then the straight line through A with direction vector \mathbf{b} has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1 \mathbf{i} + n_2 \mathbf{j} + n_3 \mathbf{k}$ has cartesian equation

$$n_1 x + n_2 y + n_3 z + d = 0$$
 where $d = -a.n$

The plane through non-collinear points A, B and C has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of
$$(\alpha, \beta, \gamma)$$
 from $n_1x + n_2y + n_3z + d = 0$ is $\frac{\left|n_1\alpha + n_2\beta + n_3\gamma + d\right|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$.

Hyperbolic functions

$$\cosh^{2} x - \sinh^{2} x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^{2} x + \sinh^{2} x$$

$$\operatorname{arcosh} x = \ln\left\{x + \sqrt{x^{2} - 1}\right\} \quad (x \ge 1)$$

$$\operatorname{arsinh} x = \ln\left\{x + \sqrt{x^{2} + 1}\right\}$$

$$\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right) \quad (|x| < 1)$$

Conics

	Ellipse	Parabola	Hyperbola	Rectangular Hyperbola
Standard Form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Parametric Form	$(a\cos\theta,b\sin\theta)$	$(at^2, 2at)$	$(a \sec \theta, b \tan \theta)$ $(\pm a \cosh \theta, b \sinh \theta)$	$\left(ct, \frac{c}{t}\right)$
Eccentricity	$e < 1$ $b^2 = a^2(1 - e^2)$	e=1	$e > 1$ $b^2 = a^2(e^2 - 1)$	$e = \sqrt{2}$
Foci	(±ae,0)	(a, 0)	(±ae, 0)	$(\pm\sqrt{2}c,\pm\sqrt{2}c)$
Directrices	$x = \pm \frac{a}{e}$	x = -a	$x = \pm \frac{a}{e}$	$x + y = \pm \sqrt{2}c$
Asymptotes	none	none	$\frac{x}{a} = \pm \frac{y}{b}$	x = 0, y = 0

Differentiation

$$f(x) f'(x)$$

$$\operatorname{arcsin} x \frac{1}{\sqrt{1-x^2}}$$

$$\operatorname{arccos} x -\frac{1}{\sqrt{1-x^2}}$$

$$\operatorname{arctan} x \frac{1}{1+x^2}$$

$$\operatorname{sinh} x \operatorname{cosh} x$$

$$\operatorname{cosh} x \sinh x$$

$$\operatorname{tanh} x \operatorname{sech}^2 x$$

$$\operatorname{arsinh} x \frac{1}{\sqrt{1+x^2}}$$

$$\operatorname{arcosh} x \frac{1}{\sqrt{x^2-1}}$$

$$\operatorname{artanh} x \frac{1}{1+x^2}$$

Integration (+ constant; a > 0 where relevant)

Arc length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$
 (cartesian coordinates)

$$s = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \, \mathrm{d}t \quad \text{(parametric form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \, ds = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$
$$= 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^{2} = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{n=1}^{n} r^{3} = \frac{1}{4} n^{2} (n+1)^{2}$$

Numerical solution of equations

The Newton-Raphson iteration for solving
$$f(x) = 0$$
: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Conics

	Parabola	Rectangular Hyperbola
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	(at ² , 2at)	$\left(ct, \frac{c}{t}\right)$
Foci	(a, 0)	Not required
Directrices	x = -a	Not required

Matrix transformations

Anticlockwise rotation through
$$\theta$$
 about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line
$$y = (\tan \theta)x$$
: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration (+ constant)

$$f(x) \qquad \int f(x) dx$$

$$\sec^2 kx \qquad \frac{1}{k} \tan kx$$

$$\tan x \qquad \ln|\sec x|$$

$$\cot x \qquad \ln|\sin x|$$

$$\csc x \qquad -\ln|\csc x + \cot x|, \quad \ln|\tan(\frac{1}{2}x)|$$

$$\sec x \qquad \ln|\sec x + \tan x|, \quad \ln|\tan(\frac{1}{2}x + \frac{1}{4}\pi)|$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A + B}{2} \cos \frac{A - B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A + B}{2} \sin \frac{A - B}{2}$$

Differentiation

f(x) f'(x)
tan kx
$$k \sec^2 kx$$

sec x $\sec x \tan x$
cot x $-\csc^2 x$
cosec x $-\csc x \cot x$

$$\frac{f(x)}{g(x)} \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^{n} = a^{n} + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^{2} + \dots + \binom{n}{r} a^{n-r}b^{r} + \dots + b^{n} \quad (n \in \mathbb{N})$$
where $\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^{n} = 1 + nx + \frac{n(n-1)}{1 \times 2}x^{2} + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \times 2 \times \dots \times r}x^{r} + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{\infty} = \frac{a}{1-r}$$
 for $|r| < 1$

Numerical integration

The trapezium rule:
$$\int_{a}^{b} y \, dx \approx \frac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$$
, where $h = \frac{b - a}{n}$

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times \text{slant height}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a+(n-1)d]$$